Non-invertible symmetries, leptons, quarks, and why multiple generations

Dual CP IHEP Workshop Colima 2024

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Based mainly on 2211.07639 with Clay Córdova (Chicago), Sungwoo Hong (KAIST), Kantaro Ohmori (Tokyo)

And forthcoming 2401.X with CC & SH

Related work in my 2204.01741, 2204.01750,

and also 2212.13193 with CC,

forthcoming 2402.X with Adam Martin (Notre Dame),

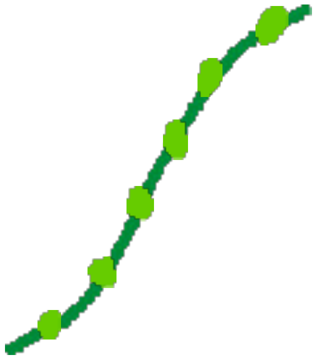
2402.X with Sam Homiller (Harvard)

Higher-form symmetries

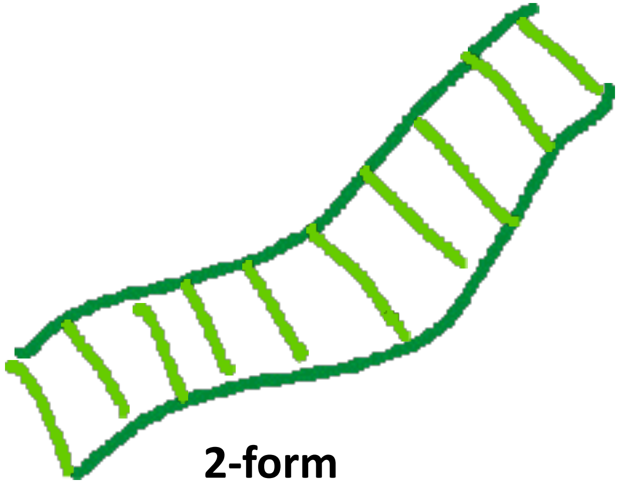
Familiar symmetries act on local operators e.g. $\psi(x) \rightarrow e^{i\alpha Q}\psi(x)$

But what about extended objects?

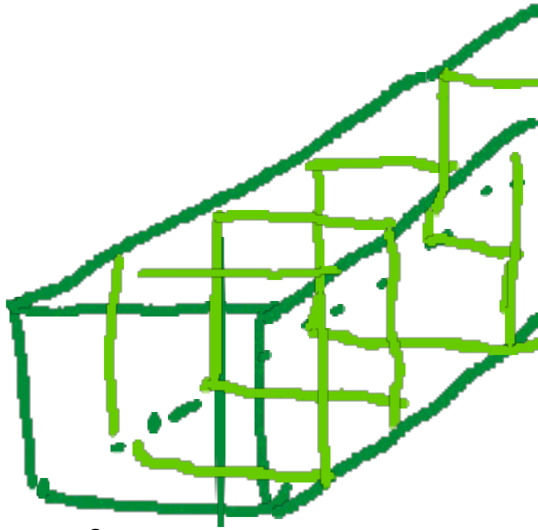
0-form symmetry
charged local
operators
e.g. particles



1-form
line operators
e.g. Wilson line



2-form
surface operators
e.g. cosmic string



3-form
volume operators
e.g. domain wall

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^{\mu\nu} = 0$$

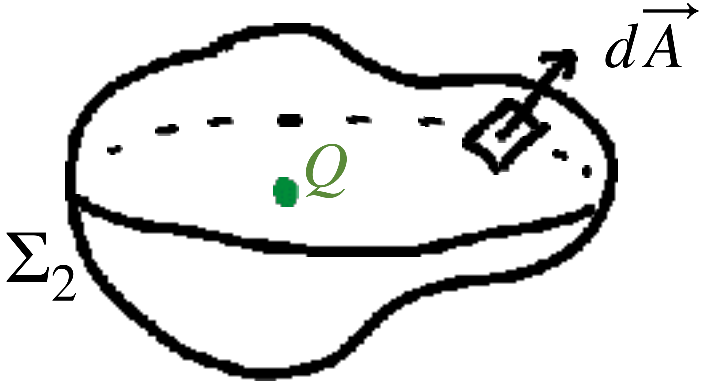
Generally $\partial_\mu J^{\mu_1 \mu_2 \cdots \mu_{p+1}} = 0$ antisymmetric

Break by adding charged operator to Lagrangian e.g. $\delta\mathscr{L} = M\bar{N}N$

Break only with the appearance of new dynamical degrees of freedom!

Generalized Global Symmetry of Electromagnetism

Recall Gauss' law: The Gaussian surface is topological and so computes an invariant charge.



$$Q_{\text{enclosed}} = \int_{\Sigma_2} \vec{E} \cdot d\vec{A} = \int_{\Sigma_2} F^{\mu\nu} dS^{\rho\sigma} \epsilon_{\mu\nu\rho\sigma}$$

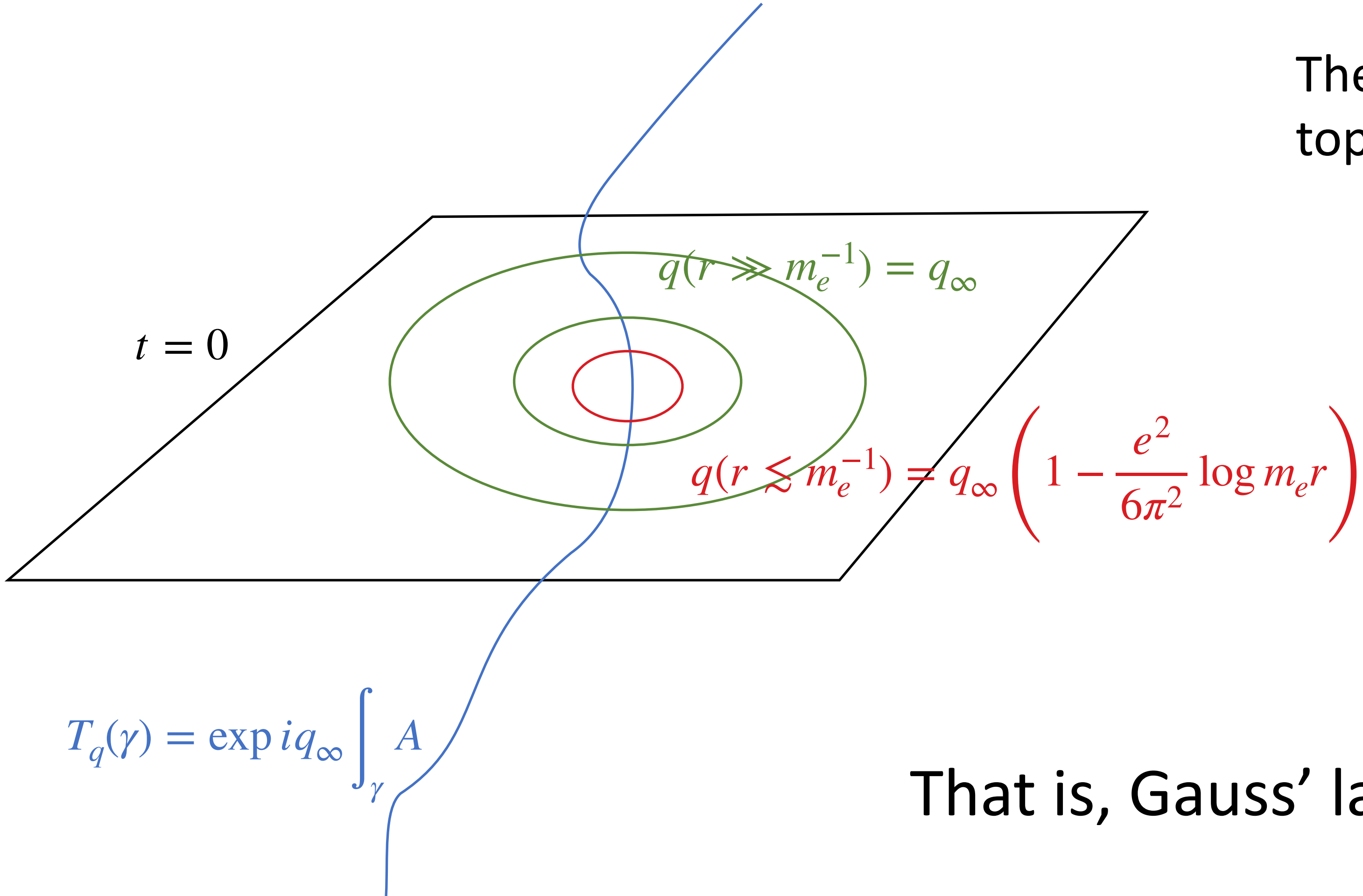
But what sort of charge is this? Clearly not a familiar Noether charge of a zero-form symmetry.

Indeed, the photon field strength is a conserved 2-form current $J_E^{\mu\nu} \sim \frac{1}{e^2} F^{\mu\nu}, \quad \partial_\mu J_E^{\mu\nu} = 0$

for the electric 1-form symmetry of pure electromagnetism.

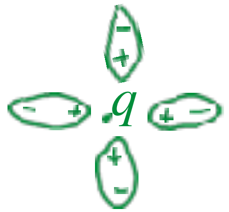
1-form symmetry and its breaking

The electric 1-form symmetry is *emergent* in the low-energy, long-distance theory



The Gaussian surface is no longer topological at short distances

Now $\partial_\mu J_E^{\mu\nu} = j^\nu$ and charges are screened



That is, Gauss' law really breaks in the UV!

Generalized global symmetries

Modern GGS understand of symmetries in terms of topological properties of operators, generalizing Noether charges/Gauss's law

In QED at distances $r \gg m_e^{-1}$ there is an electric 1-form symmetry from absence of charged matter

Similarly in U(1) gauge theory there is a magnetic 1-form symmetry $\partial_\mu J_M^{\mu\nu} = 0$ with $J_M^{\mu\nu} \sim \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ from absence of monopoles

Today's usage: Magnetic 1-form symmetry can interplay with 0-form symmetries, and teach us about interesting physics possibilities in a UV completion that provides magnetic monopoles.

Today's Goal: Model-Building Guided by Generalized Symmetries

- The Standard Model is a beautiful, yet incomplete picture
- Where to look? Start with simplest such class: Gauging some of the approximate symmetries of the SM.
- Recall this is the class of the simplest GUTs with no exotic fermions. E.g. the SU(5) approximate symmetry of the SM fermions is broken by $y_u \neq y_d \neq y_e$ (and $g_1 \neq g_2 \neq g_3$).

Why multiple generations?

One sort of answer from physics effects you can only get with multiple generations

- CP violation in CKM *would have been* a great answer if this were responsible for electroweak baryogenesis, but alas. Kuzmin, Rubakov, Shaposhnikov '85
- SM has anomaly-free $\mathbb{Z}_{2N_g}^{B+L}$ so proton not destabilized for $N_g > 1$ but lifetime very long anyway. Explained in SK '22

To me this question motivates thinking about BSM effects you can have only because $N_g > 1$, and especially interesting things that can happen for $N_g = 3$

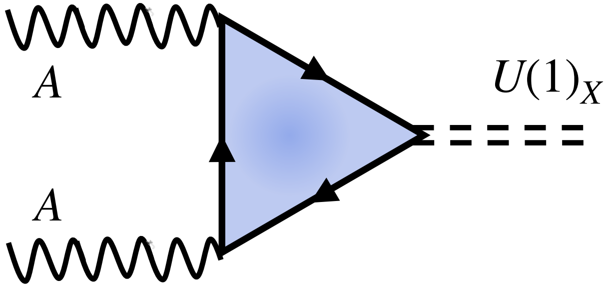
Related forthcoming SK & S. Homiller: Only for $N_g = 3$ can one write a Totally Anti-Symmetric Triplet Yukawa (TASTY) model of flavor: $y_t \epsilon_{ijk} H^i Q^j \bar{u}^k$

Recall the Standard Model

Classical Global Symmetry

$$G_{SM}^{c} = U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times U(1)_B/\mathbb{Z}_{N_c}$$

Next consider the ABJ anomalies

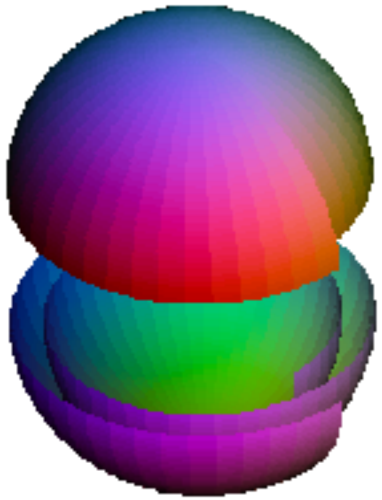


Under $U(1)_X$ transformation by α, $S \to S + i\alpha \int_{\mathscr{M}} \mathrm{tr} F\tilde{F}$

$$\partial_\mu J_X^\mu = \frac{k}{8\pi^2} \mathrm{tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$U(1)_X$ violated by instantons!

Instantons



Classical, finite-action solutions to the Yang-Mills EoM

$$A_\mu^{(1-inst)}(x) = \frac{2}{g}\frac{\rho^2}{(x-x_0)^2}\frac{\eta_{a\mu\nu}(x-x_0)^\nu J^a}{(x-x_0)^2+\rho^2}$$

Semiclassics: Sum over all saddle points

$$\int \mathcal{D}\mathscr{A} e^{-S} \simeq \sum_{\mathscr{A}=A^{(n)}+A} e^{-S_{inst}} \int \mathcal{D}A e^{-S}$$

Why? Guaranteed by a topological quantum number $\pi_3(SU(2)) = \mathbb{Z}$

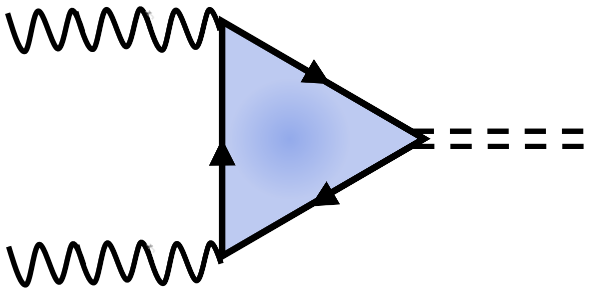
$$S_{inst} = \frac{1}{2}\int_{\mathbb{R}^4} \mathrm{tr} FF \geq \frac{1}{2}\int_{\mathbb{R}^4} \mathrm{tr} F\tilde{F} = \frac{8\pi^2}{g^2} n$$

Recall the Standard Model

Classical Global Symmetry

$$G^c_{SM} = U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau} \times U(1)_B/\mathbb{Z}_{N_c}$$

Next consider the ABJ anomalies



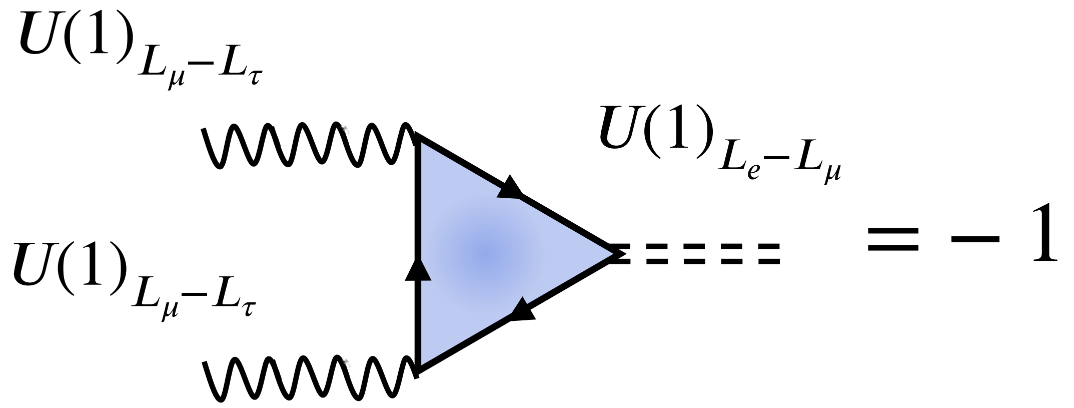
	$SU(2)^2_L$	$U(1)^2_Y$	$SU(3)^2_c$
$U(1)_B$	$N_g N_c$	$-18 N_g N_c$	0
$U(1)_{L_k}$	1	-18	0
$U(1)_L$	N_g	$-18 N_g$	0

Quantum Symmetry

$$G_{SM} = U(1)_{L_e - L_\mu} \times U(1)_{L_\mu - L_\tau} \times U(1)_{B - N_c L}/\mathbb{Z}_{N_c} \times \mathbb{Z}^L_{N_g}$$

Now let's go beyond and gauge $U(1)_{L_\mu - L_\tau}$

There's a new ABJ anomaly diagram to consider



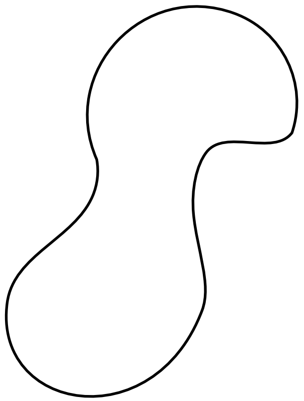
So the $L_e - L_\mu$ current is no longer conserved, $\partial_\mu J^\mu_{L_e - L_\mu} = \frac{-1}{8\pi^2} F_{L_\mu - L_\tau} \tilde{F}_{L_\mu - L_\tau}$

But $\pi_3(U(1)) = 1$, so no $U(1)$ instantons on $\mathbb{R}^4$, and indeed the S-matrix will continue to preserve $U(1)_{L_e - L_\mu}$. So what's going on—is the symmetry broken?

A subtler notion of symmetry

Modern analysis: Symmetry not fully broken; instead converted to a 'non-invertible' symmetry

Acts now both on local fields and on 't Hooft lines of the theory's magnetic one-form symmetry



$$L_\mu \to L_\mu e^{i\alpha} \qquad\qquad e^{i\oint_\gamma A_m} \to e^{i\oint_\gamma A_m + i\alpha\oint_\gamma A}$$

Zero- and one-form symmetries intertwined

Physically, this captures the fact that monopoles provide topology needed to activate Abelian instantons c.f. Callan-Rubakov

$$\int F\tilde{F} \sim \int E \cdot B \neq 0$$

van Beest, Boyle Smith, Delmastro, Komargodski, Tong '23

Can estimate breaking from loop of dynamical monopoles

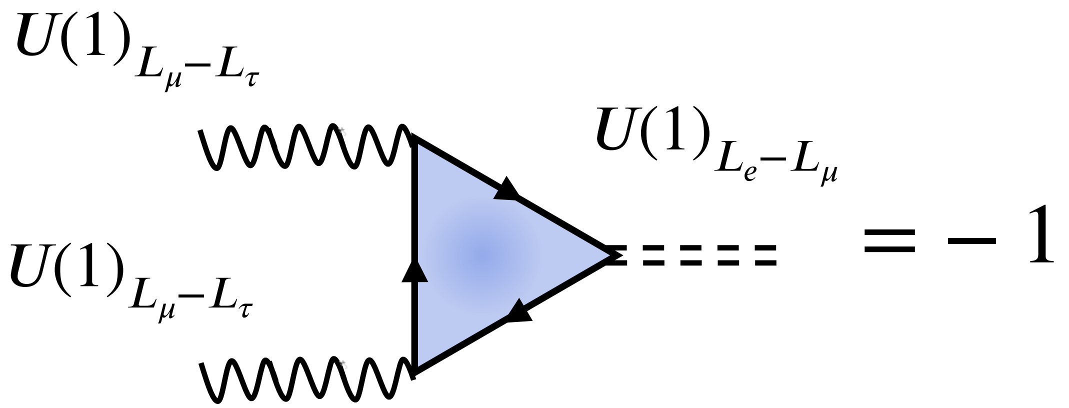
Mass $m \sim v/g$, Cutoff $\Lambda \sim gv$

$$\delta\mathscr{L} \sim \exp(-S_{mon}) \sim \exp(-m\delta t) \sim \exp(-\#/g^2)$$

[Fan, Fraser, Reece, Stout '21]

Model-building plan: Given IR theory with non-invertible symmetry, find UV embedding providing monopoles

Beyond with $Z'_{L_\mu - L_\tau}$



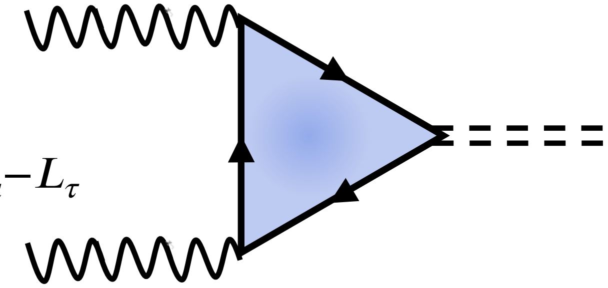
Non-invertible symmetry protects neutrino masses

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	$+1$	-1

Disallows $\left(\tilde{H}L\right)^2$

$$\exp\left(\frac{2\pi i}{3}L\right) = \exp\left(\frac{2\pi i}{3}\left((L_e - L_\mu) - (L_\mu - L_\tau)\right)\right)$$

Beyond with $Z'_{L_\mu - L_\tau}$



Non-invertible symmetry protects neutrino masses
either with or without right-handed neutrinos

	L_i	$\bar{e}_i$
$\mathbb{Z}_3^L$	$+1$	-1

Disallows $\left(\tilde{H}L\right)^2$

	L_i	$\bar{e}_i$	N
$\mathbb{Z}_3^{\tilde{L}+N}$	$+1$	-1	$+1$

Disallows $\tilde{H}LN$

Dirac masses:

	$SU(3)_H$	$U(1)_{\mu-\tau}$	$U(1)_{\tilde{L}}$	$U(1)_N$
$\mathbf{L}$	3	$\begin{pmatrix} L_e \\ L_\mu \\ L_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ +1 \\ -1 \end{pmatrix}$	$+1$	0
$\bar{\mathbf{e}}$	$\bar{3}$	$\begin{pmatrix} \bar{e} \\ \bar{\mu} \\ \bar{\tau} \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	-1	0
$\mathbf{N}$	$\bar{3}$	$\begin{pmatrix} N_e \\ N_\mu \\ N_\tau \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ +1 \end{pmatrix}$	0	$+1$

Write down charged lepton mass

$$\mathscr{L} \sim y_\tau H \mathbf{L}\bar{\mathbf{e}}$$

Classical $U(1)_N$ symmetry protects the Dirac neutrino mass $\tilde{H}\mathbf{L}\mathbf{N}$

Now turn on quantum mechanics

Fermi zero-modes

Not only boson zero-modes, but for deep topological reasons (c.f. Atiyah-Singer) instantons have *fermionic* zero-modes $i\gamma_\mu D^\mu \psi^0 = 0$

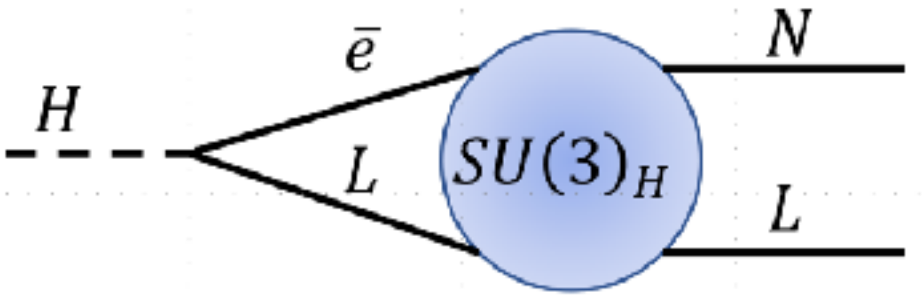
Striking, since no action cost to excite $\int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-\int \bar{\psi} i\gamma_\mu D^\mu \psi} \supset \int d\psi^0 e^{-0} = 0$

The only nonvanishing correlation functions around a background with fermi zero-modes have fermion fields in them

$$\langle \mathcal{O}\psi_1\psi_2 \rangle \supset \int d\psi_1^0 d\psi_2^0 \mathcal{O}\psi_1\psi_2 \neq 0$$

’t Hooft Vertices

Pioneering work from ’t Hooft understood that you can think of the instantons as inducing a multi-fermion interaction in the theory



Here this gives us Dirac natural Dirac neutrino masses

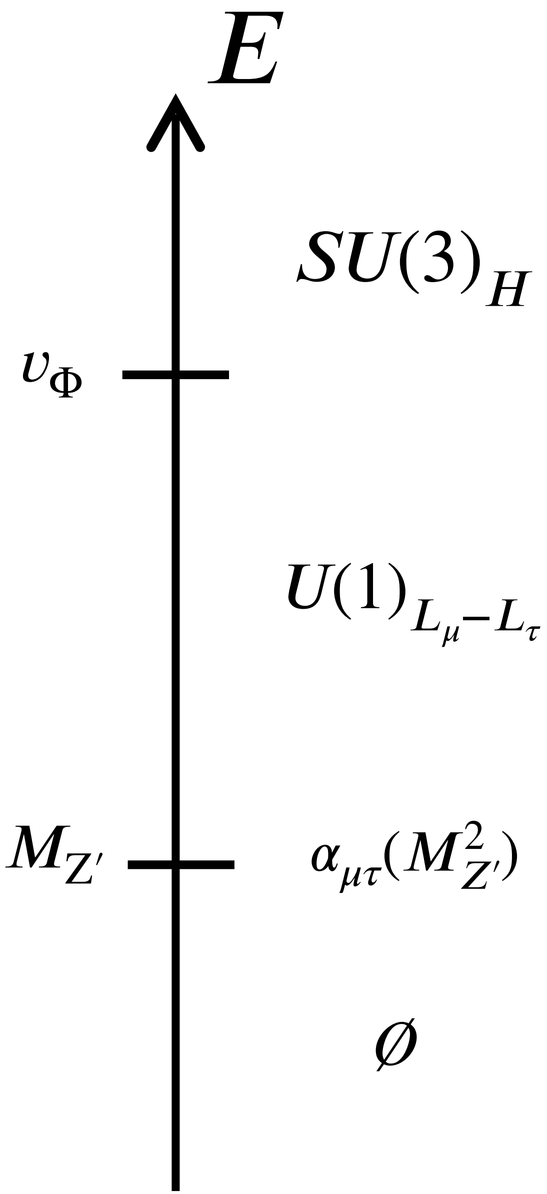
$$\mathscr{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}\mathbf{L}\mathbf{N}$$

Extremely economical

$$\mathscr{L} \sim y_\tau^\star e^{-\frac{8\pi^2}{g_H^2}} \tilde{H}\mathbf{L}\mathbf{N}$$

Run gauge coupling up $$\alpha_{\mu\tau}\left(v_\Phi^2\right)^{-1} = \alpha_{\mu\tau}\left(M_{Z'}^2\right)^{-1} - \frac{4}{3\pi}\log\frac{v_\Phi^2}{M_{Z'}^2}$$

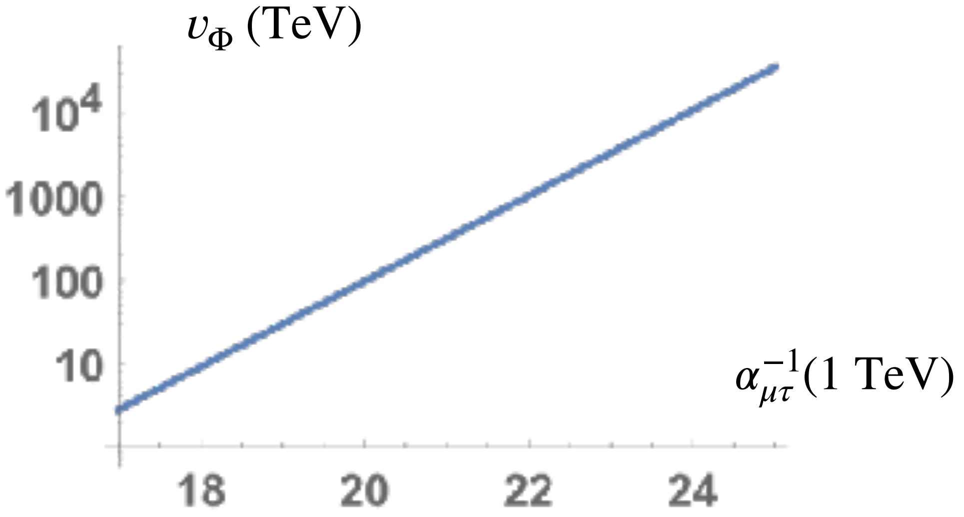
$$m_\nu \sim m_\tau \left(\frac{v_\Phi}{M_{Z'}}\right)^{4/3} \exp\frac{-\pi}{2\alpha_{\mu\tau}(M_{Z'}^2)}$$



From Z' to horizontal symmetry

Given the discovery of such a Z' for $U(1)_{L_\mu - L_\tau}$, learn the scale at which $SU(3)_H \to U(1)_{L_\mu - L_\tau}$

$$v_\Phi^2 \sim M_{Z'}^2 \left(\frac{m_\nu}{m_\tau}\right)^{3/2} \exp \frac{3\pi}{4\alpha_{\mu\tau}(M_{Z'}^2)}$$



Texture from Higgses implementing $SU(3)_H \to U(1)_{L_\mu - L_\tau} \to \emptyset$

c.f. [Alonso-Álvarez, Cline '21]

Recap: lessons from non-invertible symmetry

- Whenever there’s a zero, there should be a symmetry explanation
- Understanding a symmetry clarifies symmetry-breaking
- (Of course also further control of IR in CMT, understanding the space of 2d CFTs, understanding Callan-Rubakov, etc.)

What about the quark sector?

- If we've found something interesting purely with the leptons, shouldn't there be something interesting to find with the quarks?
- But quarks have this pesky extra $SU(3)_C$ quantum number which means you'll get thrice as many legs in a 't Hooft vertex, and generating a 12-quark interaction is not so interesting.

Require a more subtle notion (and more subtle usage) of non-invertible symmetry

What *about* the quark sector?

Don't need tiny Yukawas, but do need to explain why $\bar{\theta} \lesssim 10^{-10}$ despite $\delta_{CKM} \sim \pi/3$

Recall the 'massless up quark' solution to strong CP

$$U(1)_{PQ} \Rightarrow y_U(\Lambda_{UV}) = 0$$

so $\bar{\theta}$ is unphysical at high scales

$$\bar{u} \to \bar{u}e^{i\alpha} \Rightarrow S \to S + i\alpha \int F\tilde{F}$$

Observed nonzero up mass could be due to $SU(3)_C$ instantons which violate this symmetry

Importantly, yukawa generated by instantons does not violate CP or generate $\bar{\theta}$

Georgi & McArthur '81
Choi, Kim, Sze '88

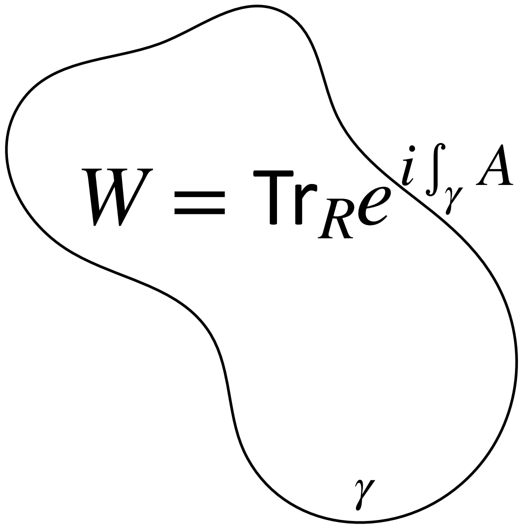
$$\mu \frac{d}{d\mu} \text{det}(m) \supset c_0 \left(\frac{8\pi^2}{g^2}\right)^6 e^{-\frac{8\pi^2}{g^2}} \mu^{n_f - 2} \text{det}(m^\dagger m) \text{Tr}(m^\dagger m)^{-1}$$

But sadly QCD instantons do not generate a large enough y_u

If only we had a way to learn from the infrared about a natural UV model with new instantons…

	Q_i	$\overline{u}_i$	$\overline{d}_i$	L_i	$\overline{e}_i$	H
$SU(3)_C$	$\mathbf{3}$	$\overline{\mathbf{3}}$	$\overline{\mathbf{3}}$	–	–	–
$SU(2)_L$	$\mathbf{2}$	–	–	$\mathbf{2}$	–	$\mathbf{2}$
$U(1)_Y$	$+1$	-4	$+2$	-3	$+6$	-3

An aside on SM one-form symmetries



Hypercharge magnetic one-form symmetry: $U(1)_m^{(1)}$

Electric one-form symmetry? We don't know!

See D. Tong '17

Certain center transformations do not act on any of the SM fields, e.g. consider $\mathbb{Z}_2 \subset SU(2)_L \times U(1)_Y$ under which $\psi \mapsto \psi\left((-1)I_L\right) e^{\pi i Y}$

So the *global structure* of the SM gauge group is

$G_{SM_q} \equiv \left(SU(3)_C \times SU(2)_L \times U(1)_Y\right)/\mathbb{Z}_q$ with q=1,2,3,6

Which has electric one-form symmetry $\mathbb{Z}_{6/q}^{(1)}$

Global structure, fractionally-charged particles, and SMEFT
SK & A. Martin coming

See also recent discussion in axion theories by Reece; Choi, Forslund, Lam, Shao; Cordova, Hong, Wang

Fractional instantons to the rescue

Non-invertible symmetry is more general than just $U(1)$ gauge theory!

- For $U(1)$ there are instantons valued in $\mathbb{Z}$, but they don't exist on $\mathbb{R}^4$
- For $SU(N)$ there are instantons valued in $\mathbb{Z}$ on $\mathbb{R}^4$
- For $SU(N)/\mathbb{Z}_N$ there are instantons valued in $\mathbb{Z}/N$, but only the $\mathbb{Z}$ -valued ones exist on $\mathbb{R}^4$

In the quark sector, specifically because we have $N_c = N_g$, we can consider a horizontal quark gauge symmetry with nontrivial global structure!

$$\left(SU(3)_C \times U(1)_{B_1+B_2-2B_3}\right)/\mathbb{Z}_3$$

Fractional instanton analysis on $S^2 \times S^2$

See Anber, Hong, Son 2109.03245

Turn on general allowed magnetic fluxes (background 2-form fields for the magnetic 1-form symmetry) and calculate instanton numbers

$$Q_c = \frac{N_c - 1}{N_c} \int_{M_4 = M_2 \times \Sigma_2} \frac{w_2 \wedge w_2}{2} = \frac{N_c - 1}{N_c} \oint_{M_2} w_2 \oint_{\Sigma_2} w_2 = m_1 m_2 \left(1 - \frac{1}{N_c} \right)$$

$$Q_H = \frac{1}{8\pi^2} \int H_2 \wedge H_2 = s_1 s_2$$

Then compute Dirac indices of fermions

$$I_{\psi_i} = n_{\psi_i} T_{\psi_i} Q_c + \mathrm{dim}_{\psi_i} n_{\psi_i} q_{\psi_i}^2 Q_H$$

And find anomaly coefficients for each $U(1)_{\mathrm{global}}[CH]^2$

Non-invertible symmetry

Now with non-trivial global structure, due to the extra fractional instantons, there is again a $\mathbb{Z}_3^{\tilde{B}+d}$ non-invertible symmetry which protects the down quark yukawas

	Q_i	$\overline{u}_i$	$\overline{d}_i$
$\mathbb{Z}_3^{\tilde{B}+d}$	$+1$	-1	$+1$

Non-abelian horizontal symmetry

- The non-trivial global structure is even more striking promoting to a $SU(3)_H$ symmetry of quarks mirroring the lepton theory
- The $SU(3)_H$ assignment is the same as the $SU(3)_C$ quantum number!
- Again we see that the gauge group can be $\left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$

	$SU(3)_c$	$SU(3)_H$
$\mathbf{Q}$	3	3
$\bar{\mathbf{u}}$	$\bar{3}$	$\bar{3}$
$\bar{\mathbf{d}}$	$\bar{3}$	$\bar{3}$

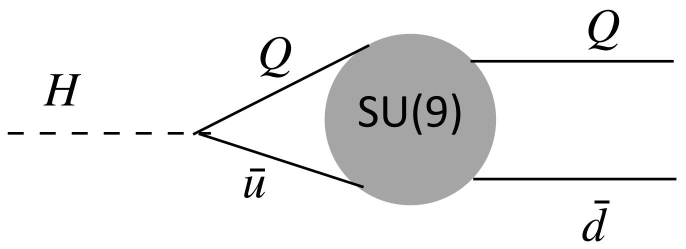
Color-flavor unification!

This all points to a nice $SU(9)$ 'unified' theory in which the colors and flavors of the quarks are placed together into the fundamental

	$SU(9)$
$\mathbf{Q}$	9
$\bar{\mathbf{u}}$	$\bar{9}$
$\bar{\mathbf{d}}$	$\bar{9}$

$$\mathscr{L} = y_t \tilde{H} Q \bar{u}$$

So $\bar{\theta}$ manifestly unphysical in far UV, but then instantons generate



$$\mathscr{L} \sim y_t^{\star} e^{-2\pi/\alpha_9(\Lambda_9)} H Q \bar{d}$$

Color-flavor embedding

See Csaki, Murayama ’98 for good discussion

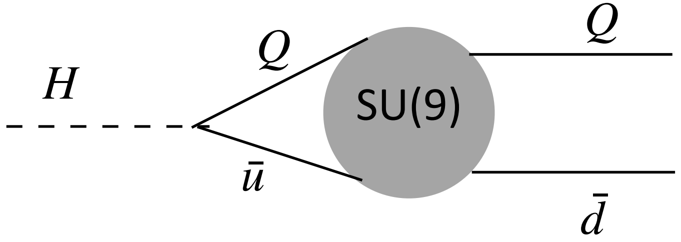
This ‘special embedding’ $SU(9) \to \left(SU(3)_C \times SU(3)_H\right)/\mathbb{Z}_3$ has a non-trivial ‘index of embedding’: the fundamental 9 branches to the $(3,3)$ so the Dynkin index changes non-trivially $k = \mu_{IR}/\mu_{UV} = 3$

So the $SU(9)$ theory has ‘extra’ instantons that the IR theory does not: a fermion has $k = 3$ times as many zero-modes in the $SU(3)_C$ instanton background, so we must interpret this as a 3-instanton of the $SU(9)$ theory

Matching the instanton actions implies a non-trivial matching of the gauge coupling across Λ_9, as $e^{-\frac{8\pi^2}{\alpha_{IR}}} = e^{-k\frac{8\pi^2}{\alpha_{UV}}}$, so $\alpha_9(\Lambda_9) = 3\alpha_C(\Lambda_9)$

Need a better estimate of instanton effects

$y_b/y_t \sim 1/40$ is not so small that we can ignore the polynomial prefactor

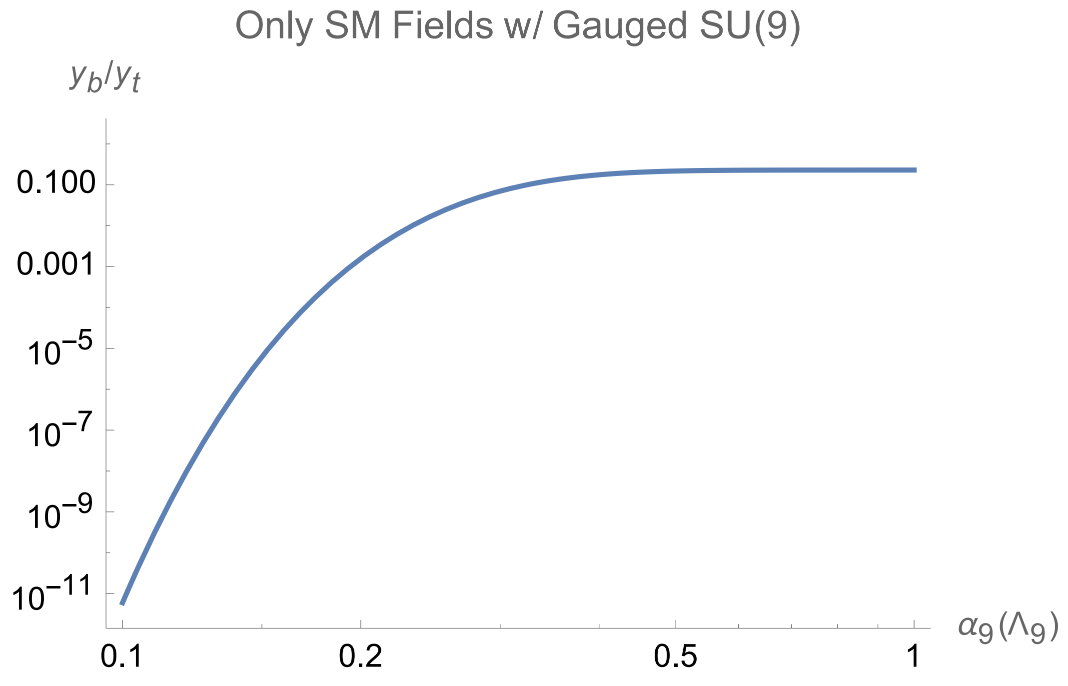


$$\sim \int_{1-inst} \mathscr{D}A\mathscr{D}\phi_i\mathscr{D}\psi_i \; H\mathbf{Q}\bar{\mathbf{d}} \; e^{-S_{gauge}-\int \mathscr{L}_{int}}$$

Thankfully 't Hooft taught us how to do this in 1976. Must integrate over all the zero-modes of the 1-inst solution.

$$A_\mu(x) = \frac{2}{g}\frac{\rho^2}{(x-x_0)^2}\frac{\eta_{a\mu\nu}(x-x_0)^\nu J^a}{(x-x_0)^2+\rho^2}$$

As well as quadratic fluctuations for any charged scalar fields, and solve for the charged fermion zero-modes



Generating CKM

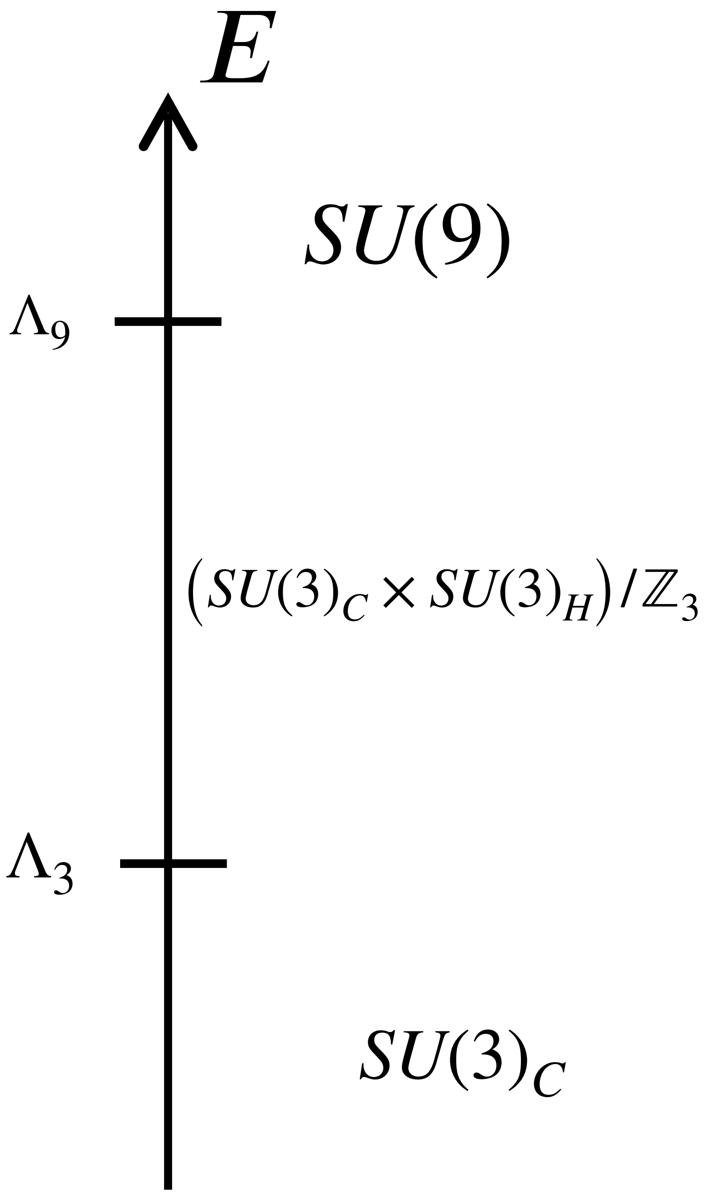
Must generate δ_{CKM} without upsetting $\bar{\theta} = 0$, but want to factorize problems since top-down flavor model-building is hard.

In particular with H we have $\mathscr{L} \supset y_t \tilde{H} Q \bar{u}$ so get

$y_u = y_t 1^a{}_b + \ldots$ breaking effects

and the quarks don't want this structure!

Probably need to upgrade $H \to H^a{}_b$ to get flavor correct. Can we understand how to solve strong CP without getting the details right?



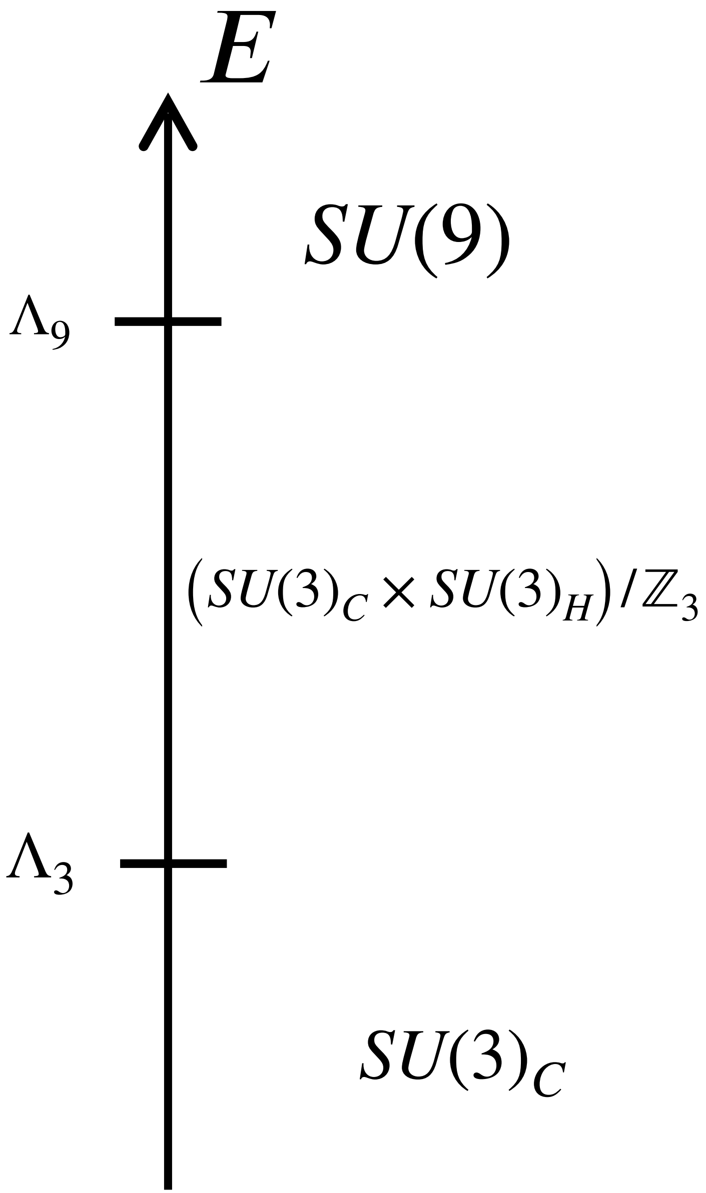
Generating CKM

Idea: Communicating flavor-breaking $\langle \Sigma^a{}_b \rangle$ through gauged flavor symmetry lets you generate *hermitian* yukawas

$\bar{\theta} = \arg\det e^{-i\theta} y_u y_d$ automatically zero

But so long as $V(\Sigma)$ breaks CP, can communicate a phase to CKM

$$\delta_{CKM} \propto \arg\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right) \neq 0$$



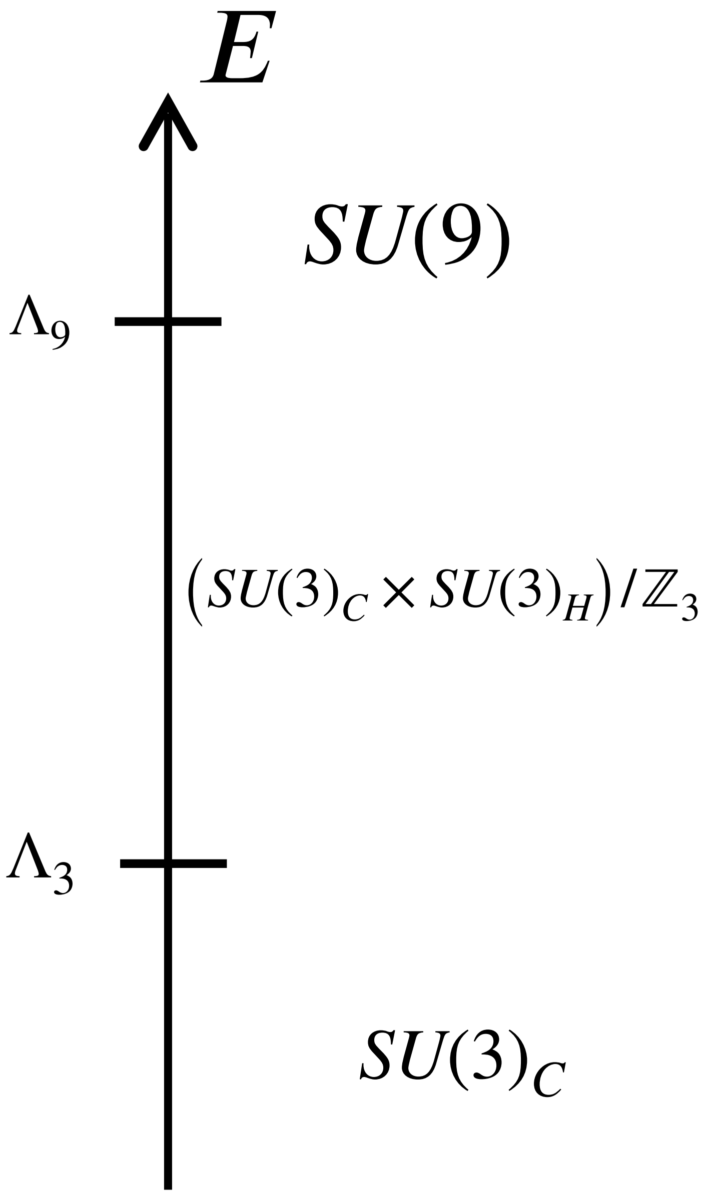
Generating CKM

$$\delta_{CKM} \propto \arg\det\left(\left[y_u^\dagger y_u, y_d^\dagger y_d\right]\right) \neq 0$$

One more wrinkle: Must treat $\bar{u}, \bar{d}$ differently so they don't commute in flavor space.

Use some extra interactions of, say, $\bar{d}$ with Σ, e.g.

$$\mathscr{L} \supset \bar{d}\tilde{d}\chi + \tilde{d}\Sigma^2\tilde{d}^\dagger$$

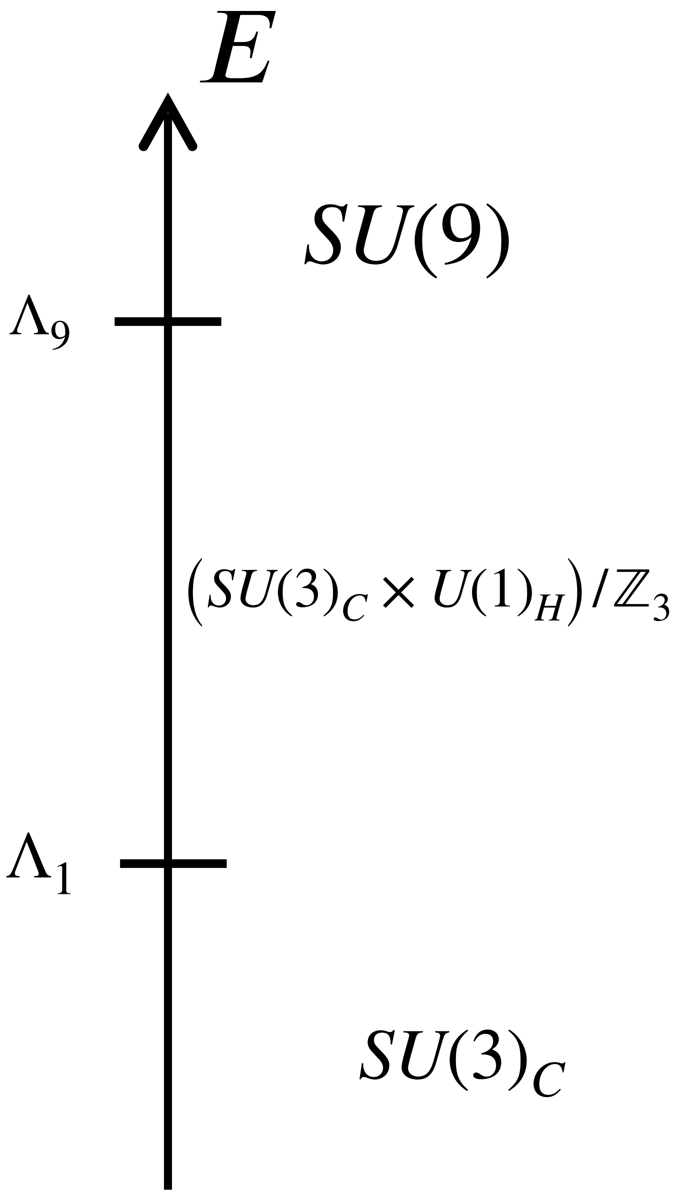


Abelian Z’ also promising

In the case of $\left(SU(3)_C \times U(1)_H\right)/\mathbb{Z}_3$ the new gauge boson (and new non-invertible symmetry) can observationally appear much sooner

And furthermore maybe don’t need as large Higgs representations to do breaking, so less suppression of instanton density

But need to more clearly understand generating flavor texture in this scheme



Conclusions

- Models you care about have generalized symmetries and understanding them can be useful
  - Learn more about UV than you might expect since their violation requires new degrees of freedom
- Theories of gauged horizontal symmetries can have nonperturbative solutions to infrared naturalness issues, and $N_g = 3$ can be special
  - Toward the IR, need to study these models in more detail with fully realistic Higgsing down to the SM
  - Toward the UV, $SU(3) \times SU(9) \subset SU(12)$ in a flavor-twisted Pati-Salam model with a common explanation for m_ν and $\bar{\theta}$

This is only the start; what more do we have to learn from subtler notions of symmetries in field theories?